CHAPTER 4

LINEAR EQUATIONS IN TWO VARIABLES

(A) Main Concepts and Results

An equation is a statement in which one expression equals to another expression. An equation of the form $ax + by + c = 0$, where a, b and c are real numbers such that $a \neq 0$ and $b \neq 0$, is called a linear equation in two variables. The process of finding solution(s) is called solving an equation.

The solution of a linear equation is not affected when

(i) the same number is added to (subtracted from) both sides of the equation,

(ii) both sides of the equation are multiplied or divided by the same non-zero number.

Further, a linear equation in two variables has infinitely many solutions. The graph of every linear equation in two variables is a straight line and every point on the graph (straight line) represents a solution of the linear equation. Thus, every solution of the linear equation can be represented by a unique point on the graph of the equation. The graphs of $x = a$ and $y = a$ are lines parallel to the y-axis and x-axis, respectively.

(B) Multiple Choice Questions

Write the correct answer:

Sample Question 1 : The linear equation $3x - y = x - 1$ has :

(A) A unique solution (B) Two solutions

(C) Infinitely many solutions (D) No solution

Solution : Answer (C)

Sample Question 2 : A linear equation in two variables is of the form $ax + by + c = 0$, where

(A) $a \neq 0, b \neq 0$ (B) $a = 0, b \neq 0$ (C) $a \neq 0, b = 0$ (D) $a = 0, c = 0$

Solution : Answer (A)

Sample Question 3 : Any point on the y-axis is of the form

(A) $(x, 0)$ (B) (x, y) (C) $(0, y)$ (D) (y, y)

Solution : Answer (C)

EXERCISE 4.1

Write the correct answer in each of the following :

1. The linear equation $2x - 5y = 7$ has

(A) A unique solution (B) Two solutions

(C) Infinitely many solutions (D) No solution

2. The equation $2x + 5y = 7$ has a unique solution, if x, y are :

(A) Natural numbers (B) Positive real numbers

(C) Real numbers (D) Rational numbers

3. If $(2, 0)$ is a solution of the linear equation $2x + 3y = k$, then the value of k is

(A) 4 (B) 6 (C) 5 (D) 2

4. Any solution of the linear equation $2x + 0y + 9 = 0$ in two variables is of the form

(A) $(-\frac{9}{2}, m)$ (B) $(n, -\frac{9}{2})$

(C) $(0, -\frac{9}{2})$ (D) $(-9, 0)$

5. The graph of the linear equation $2x + 3y = 6$ cuts the y-axis at the point

(A) $(2, 0)$ (B) $(0, 3)$ (C) $(3, 0)$ (D) $(0, 2)$

6. The equation $x = 7$, in two variables, can be written as

(A) $1 . x + 1 . y = 7$ (B) $1. x + 0. y = 7$

(C) $0 . x + 1 . y = 7$ (D) $0 . x + 0 . y = 7$

7. Any point on the x-axis is of the form

(A) (x, y) (B) $(0, y)$ (C) $(x, 0)$ (D) (x, x)

8. Any point on the line $y = x$ is of the form

(A) (a, a) (B) $(0, a)$ (C) $(a, 0)$ (D) $(a, -a)$

9. The equation of x-axis is of the form

(A) $x = 0$ (B) $y = 0$ (C) $x + y = 0$ (D) $x = y$

10. The graph of $y = 6$ is a line

(A) parallel to x-axis at a distance 6 units from the origin

(B) parallel to y-axis at a distance 6 units from the origin

(C) making an intercept 6 on the x-axis.

(D) making an intercept 6 on both the axes.

11. $x = 5$, $y = 2$ is a solution of the linear equation

(A) $x + 2y = 7$ (B) $5x + 2y = 7$ (C) $x + y = 7$ (D) $5x + y = 7$

12. If a linear equation has solutions $(-2, 2)$, $(0, 0)$ and $(2, -2)$, then it is of the form

(A) $y - x = 0$ (B) $x + y = 0$

(C) $-2x + y = 0$ (D) $-x + 2y = 0$

13. The positive solutions of the equation $ax + by + c = 0$ always lie in the

(A) 1st quadrant (B) 2nd quadrant

(C) 3rd quadrant (D) 4th quadrant

14. The graph of the linear equation $2x + 3y = 6$ is a line which meets the x-axis at the point

(A) $(0, 2)$ (B) $(2, 0)$ (C) $(3, 0)$ (D) $(0, 3)$

15. The graph of the linear equation $y = x$ passes through the point

(A) $\left(\frac{3}{2}, \frac{-3}{2}\right)$ (B) $\left(0, \frac{3}{2}\right)$ (C) $(1, 1)$ (D) $\left(\frac{-1}{2}, \frac{1}{2}\right)$

16. If we multiply or divide both sides of a linear equation with a non-zero number, then the solution of the linear equation :

(A) Changes

(B) Remains the same

(C) Changes in case of multiplication only

(D) Changes in case of division only

17. How many linear equations in x and y can be satisfied by $x = 1$ and $y = 2$?

(A) Only one (B) Two (C) Infinitely many (D) Three

18. The point of the form (a, a) always lies on :

(A) x-axis (B) y-axis

(C) On the line $y = x$ (D) On the line $x + y = 0$

19. The point of the form $(a, -a)$ always lies on the line

(A) $x = a$ (B) $y = -a$ (C) $y = x$ (D) $x + y = 0$

(C) Short Answer Questions with Reasoning

Sample Question 1 : Write whether the following statements are **True** or **False**? Justify your answers.

(i) $ax + by + c = 0$, where a, b and c are real numbers, is a linear equation in two variables.

(ii) A linear equation $2x + 3y = 5$ has a unique solution.

(iii) All the points $(2, 0)$, $(-3, 0)$, $(4, 2)$ and $(0, 5)$ lie on the x-axis.

(iv) The line parallel to the y-axis at a distance 4 units to the left of y-axis is given by the equation $x = -4$.

(v) The graph of the equation $y = mx + c$ passes through the origin.

Solution :

(i) False, because $ax + by + c = 0$ is a linear equation in two variables if both a and b are non-zero.

(ii) False, because a linear equation in two variables has infinitely many solutions.

(iii) False, the points $(2, 0)$, $(-3, 0)$ lie on the x-axis. The point $(4, 2)$ lies in the first quadrant. The point $(0, 5)$ lies on the y-axis.

(iv) True, since the line parallel to y-axis at a distance a units to the left of y-axis is given by the equation $x = -a$.

(v) False, because $x = 0, y = 0$ does not satisfy the equation.

Sample Question 2 : Write whether the following statement is **True** or **False**? Justify your answer.

The coordinates of points given in the table :

x	0	1	2	3	4
y	2	4	6	8	10

represent some of the solutions of the equation $2x + 2 = y$.

Solution : True, since on looking at the coordinates, we observe that each y-coordiante is two units more than double the x-coordinate.

EXERCISE 4.2

Write whether the following statements are True or False? Justify your answers :

1. The point (0, 3) lies on the graph of the linear equation $3x + 4y = 12$.

2. The graph of the linear equation $x + 2y = 7$ passes through the point (0, 7).

3. The graph given below represents the linear equation $x + y = 0$.

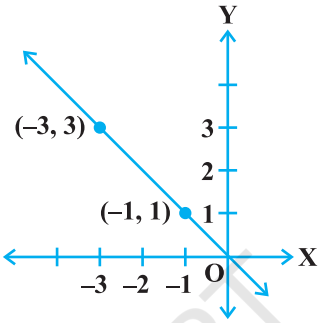

Fig. 4.1

4. The graph given below represents the linear equation $x = 3$ (see Fig. 4.2).

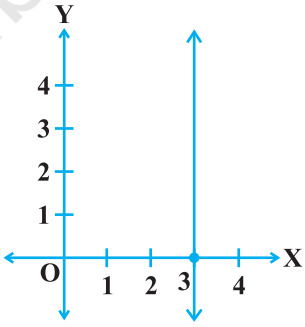

Fig. 4.2

5. The coordinates of points in the table:

x	0	1	2	3	4
y	2	3	4	–5	6

represent some of the solutions of the equation $x - y + 2 = 0$.

6. Every point on the graph of a linear equation in two variables does not represent a solution of the linear equation.

7. The graph of every linear equation in two variables need not be a line.

(D) Short Answer Questions

Sample Question 1 : Find the points where the graph of the equation $3x + 4y = 12$ cuts the x-axis and the y-axis.

Solution : The graph of the linear equation $3x + 4y = 12$ cuts the x-axis at the point where $y = 0$. On putting $y = 0$ in the linear equation, we have $3x = 12$, which gives $x = 4$. Thus, the required point is (4, 0).

The graph of the linear equation $3x + 4y = 12$ cuts the y-axis at the point where $x = 0$. On putting $x = 0$ in the given equation, we have $4y = 12$, which gives $y = 3$. Thus, the required point is (0, 3).

Sample Question 2 : At what point does the graph of the linear equation $x + y = 5$ meet a line which is parallel to the y-axis, at a distance 2 units from the origin and in the positive direction of x-axis.

Solution : The coordinates of the points lying on the line parallel to the y-axis, at a distance 2 units from the origin and in the positive direction of the x-axis are of the form $(2, a)$. Putting $x = 2$, $y = a$ in the equation $x + y = 5$, we get $a = 3$. Thus, the required point is (2, 3).

Sample Question 3 : Determine the point on the graph of the equation $2x + 5y = 20$ whose x-coordinate is $\frac{5}{2}$ times its ordinate.

Solution : As the x-coordinate of the point is $\frac{5}{2}$ times its ordinate, therefore, $x = \frac{5}{2}y$.

Now putting $x = \frac{5}{2}y$ in $2x + 5y = 20$, we get, $y = 2$. Therefore, $x = 5$. Thus, the required point is (5, 2).

Sample Question 4 : Draw the graph of the equation represented by the straight line which is parallel to the x-axis and is 4 units above it.

Solution : Any straight line parallel to x-axis is given by $y = k$, where k is the distance of the line from the x-axis. Here $k = 4$. Therefore, the equation of the line is $y = 4$. To draw the graph of this equation, plot the points (1, 4) and (2, 4) and join them. This is the required graph (see Fig. 4.3).

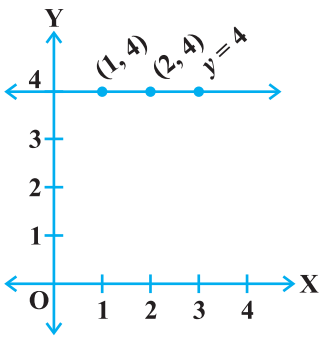

Fig. 4.3

EXERCISE 4.3

1. Draw the graphs of linear equations
$y = x$ and $y = -x$ on the same cartesian plane.
What do you observe?

2. Determine the point on the graph of the linear equation $2x + 5y = 19$, whose ordinate is $1\frac{1}{2}$ times its abscissa.

3. Draw the graph of the equation represented by a straight line which is parallel to the x-axis and at a distance 3 units below it.

4. Draw the graph of the linear equation whose solutions are represented by the points having the sum of the coordinates as 10 units.

5. Write the linear equation such that each point on its graph has an ordinate 3 times its abscissa.

6. If the point (3, 4) lies on the graph of $3y = ax + 7$, then find the value of a.

7. How many solution(s) of the equation $2x + 1 = x - 3$ are there on the :

(i) Number line (ii) Cartesian plane

8. Find the solution of the linear equation $x + 2y = 8$ which represents a point on

(i) x-axis (ii) y-axis

9. For what value of c, the linear equation $2x + cy = 8$ has equal values of x and y for its solution.

10. Let y varies directly as x. If $y = 12$ when $x = 4$, then write a linear equation. What is the value of y when $x = 5$?

(E) Long Answer Questions

Sample Question 1 : Draw the graph of the linear equation $2x + 3y = 12$. At what points, the graph of the equation cuts the x-axis and the y-axis?

Solution : The given equation is $2x + 3y = 12$. To draw the graph of this equation, we need at least two points lying on the graph.

From the equation, we have $y = \frac{12 - 2x}{3}$

For $x = 0$, $y = 4$, therefore, (0, 4) lies on the graph.

For $y = 0$, $x = 6$, therefore, (6, 0) lies on the graph.

Now plot the points A (0, 4) and B (6, 0) and join them (see Fig 4.4), to get the line AB. Line AB is the required graph.

You can see that the graph (line AB) cuts the x-axis at the point (6, 0) and the y-axis at the point (0, 4).

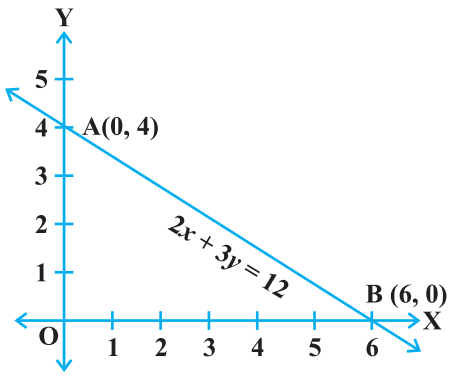

Fig. 4.4

Sample Question 2 : The following values of x and y are thought to satisfy a linear equation :

x	1	2
y	1	3

Draw the graph, using the values of x, y as given in the above table.

At what point the graph of the linear equation

(i) cuts the x-axis. (ii) cuts the y-axis.

Solution : From the table, we get two points A (1, 1) and B (2, 3) which lie on the graph of the linear equation. Obviously, the graph will be a straight line. So, we first plot the points A and B and join them as shown in the Fig 4.5.

From the Fig 4.5, we see that the graph cuts the x-axis at the point $\left(\frac{1}{2}, 0\right)$ and the y-axis at the point $(0, -1)$.

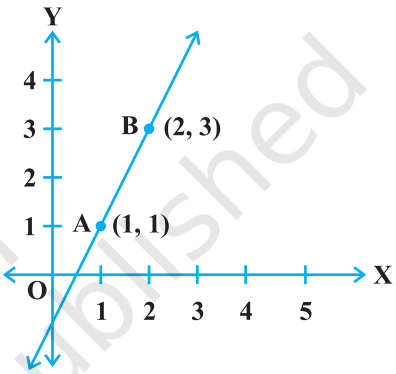

Fig. 4.5

Sample Question 3 : The Autorikshaw fare in a city is charged Rs 10 for the first kilometer and @ Rs 4 per kilometer for subsequent distance covered. Write the linear equation to express the above statement. Draw the graph of the linear equation.

Solution : Let the total distance covered be x km and the fare charged Rs y. Then for the first km, fare charged is Rs 10 and for remaining $(x - 1)$ km fare charged is Rs 4 $(x - 1)$.

Therefore, $y = 10 + 4(x - 1) = 4x + 6$

The required equation is $y = 4x + 6$. Now, when $x = 0$, $y = 6$ and when $x = -1$, $y = 2$. The graph is given in Fig 4.6.

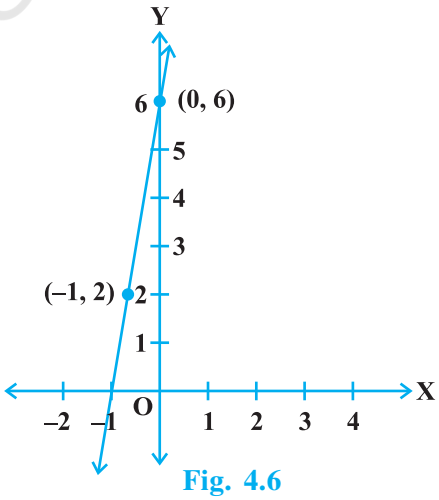

Fig. 4.6

Sample Question 4 : The work done by a body on application of a constant force is the product of the constant force and the distance travelled by the body in the direction of force. Express this in the form of a linear equation in two variables and draw its

graph by taking the constant force as 3 units. What is the work done when the distance travelled is 2 units. Verify it by plotting the graph.

Solution: Work done = (constant force) × (distance)

= 3 × (distance),

i.e., $y = 3x$, where y (units) is the work done and x (units) is the distance travelled. Since $x = 2$ units (given), therefore, work done = 6 units. To plot the graph of the linear equation $y = 3x$, we need at least two solutions of the equation. We see that $x = 0$, $y = 0$ satisfies the given equation also $x = 1$, $y = 3$ satisfies the equation.

Now we plot the points A (0, 0) and B (1, 3) and join AB (see Fig. 4.7). The graph of the equation is a straight line. [We have not shown the whole line because work done cannot be negative].

To verify from the graph, draw a perpendicular to the x-axis at the point (2, 0) meeting the graph at the point C. Clearly the coordinates of C are (2, 6). It means that the work done is 6 units.

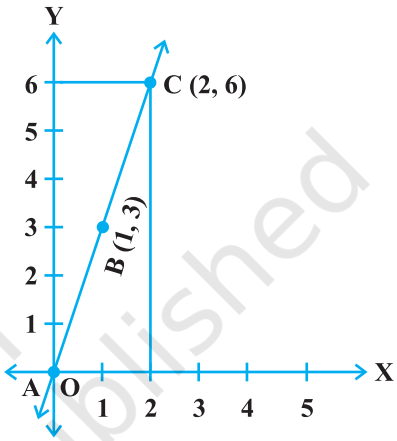

Fig. 4.7

EXERCISE 4.4

1. Show that the points A (1, 2), B (– 1, – 16) and C (0, – 7) lie on the graph of the linear equation $y = 9x – 7$.

2. The following observed values of x and y are thought to satisfy a linear equation. Write the linear equation :

x	6	–6
y	–2	6

Draw the graph using the values of x, y as given in the above table.

At what points the graph of the linear equation

(i) cuts the x-axis (ii) cuts the y-axis

3. Draw the graph of the linear equation $3x + 4y = 6$. At what points, the graph cuts the x-axis and the y-axis.

4. The linear equation that converts Fahrenheit (F) to Celsius (C) is given by the relation

$$C=\frac{5F-160}{9}$$

(i) If the temperature is 86°F, what is the temperature in Celsius?

(ii) If the temperature is 35°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 0°C what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(iv) What is the numerical value of the temperature which is same in both the scales?

5. If the temperature of a liquid can be measured in Kelvin units as x°K or in Fahrenheit units as y°F, the relation between the two systems of measurement of temperature is given by the linear equation

$$y = \frac{9}{5}(x - 273) + 32$$

(i) Find the temperature of the liquid in Fahrenheit if the temperature of the liquid is 313°K.

(ii) If the temperature is 158° F, then find the temperature in Kelvin.

6. The force exerted to pull a cart is directly proportional to the acceleration produced in the body. Express the statement as a linear equation of two variables and draw the graph of the same by taking the constant mass equal to 6 kg. Read from the graph, the force required when the acceleration produced is (i) 5 m/sec^2, (ii) 6 m/sec^2.